# 5

# Particle Accelerators

## 5.1 Introduction

With the possibility of transmutation of elements by bombarding nuclei with sub-atomic particles, the search for particles with higher and higher energies began. This led to the development of a series of powerful particle-accelerators. An ***accelerator*** is a device used to accelerate and thus impart high kinetic energy to charged particles and produce thereby a beam of high energy particles. Apart from transmutation, such high energy particle-beams are needed to understand the nuclear structure dynamics of nuclear reactions and are also used in the study of nuclear scattering and other nuclear properties, including the nature of nuclear forces. In fact, to a nuclear physicist, an accelerator is like a microscope to a biologist and a telescope to an astronomer. Accelerators are also finding applications in medicine as a tool of radiation therapy for the treatment of cancer, and also in agriculture.

The *essentials* of an accelerator set-up are: (a) an ***ion source*** whence the particles are injected into the accelerator, (b) a ***collimator*** to collimate the particles into a parallel beam, (c) an ***arrangement for acceleration*** of particles by electric and magnetic fields along specific trajectories and (d) ***deflectors*** to deflect the accelerated beam of particles to the targets. Each of these components is more or less equally important.

A number of accelerating machines has been devised since 1930's and continuously improved to impart energies from few MeV to several TeV. As the maximum energy increases, the cost of construction and maintenance also rises to become almost near astronomical in figures. Different types of accelerators have been constructed to accelerate protons, deuterons, α-particles, electrons etc. that resulted in the discovery of a number of elementary particles.

The machines that can produce a high dc voltage fall under the general denomination of accelerators. Earlier accelerators were in fact only high voltage generators.

• ***But why are high energies needed to study the nuclear structure?*** The de Broglie wavelength $\lambda$ of a particle of momentum $p$ is $\lambda = h/p$ where $h$ is Planck's constant. It is known from optics that to see the structure in detail of an object of linear dimensions $d$, the wavelength $\lambda$ must satisfy $\lambda \leq d$. So, for this $\lambda$, the momentum, $p \geq h/d$. Thus, to '*see*' the nuclear particles, high momenta and hence high energies are required. For instance, for $d = 10^{-15}$ m and with protons as probes, calculations show that the kinetic energy should be about 20 MeV. Thus it must be produced *artificially*.

The contents of this chapter would essentially be the descriptions of the construction, the principle of action and applications of a number of present-day-accelerators.

## 5.2 van de Graaff electrostatic generator

***van de Graaff electrostatic generator***, an *electrostatic* machine, developed in 1931 by van de Graaff at MIT, USA, is based on the ***principle*** that if a charged conductor is brought in internal contact with another conductor, which is hollow, all of its charge gets transferred to the hollow conductor, no matter how high is its potential. So, by repeated transfer of charge, potential of the hollow conductor can be raised to a very high desired value.

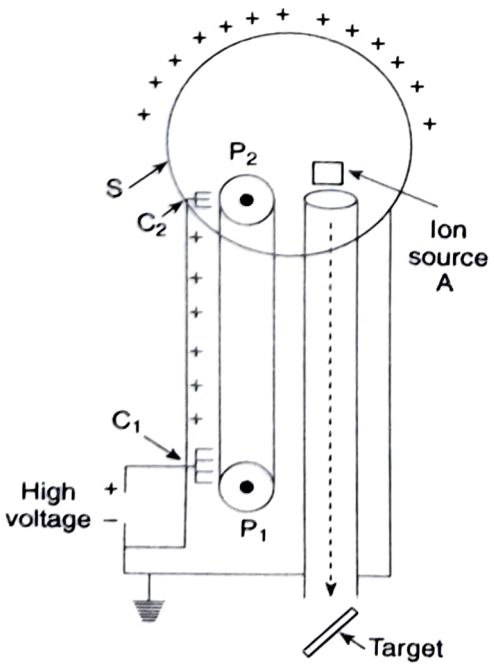

**Fig. 5.1** van de Graaff generator

**Construction :** It consists of a *giant hollow metallic sphere* $S$ (Fig. 5.1), mounted on a long insulated support. The hollow conductor is called *high voltage terminal* or *dome*. An *endless belt of insulating material* such as paper, rubber, silk, plastics etc. runs by a motor vertically between two frictionless pulleys $P_1$ and $P_2$. $P_2$ is inside the dome and $P_1$ mounted on grounded end. $C_1$ and $C_2$ are two sharp pointed combs, called respectively the *spray comb* and *charge-collection comb*. Comb $C_1$ charges the belt with charge sprayed by its sharp needles, close to the belt at $P_1$. $C_1$ is connected to a D.C. power supply at a high voltage of about 10 kV. Comb $C_2$ is placed near the belt $P_2$ inside the high voltage dome for collecting and passing on the charge to the sphere $S$ by a corona discharge.

The belt returns to the bottom after delivering the charge and is recharged at $C_1$ to repeat the process. The charge thus accumulates rapidly on the sphere where the potential rises ($V = Q/C$), the ceiling of accumulation being set by the rate of leakage through the supporting columns of the sphere or the surrounding air medium. Sharp points of $C_1$ attracts negative electrons and repel positive ions towards the fast moving belt which gets positively charged. As the positively charged belt moves up in the dome, it picks up electrons from $C_2$ and the belt is neutralized.

The problem of leaking of charge puts an upper limit to high voltage generation. To build up the potential by increasing the insulation of the surrounding air, the sphere is usually enclosed in a *steel pressure tank* (not shown) containing $N_2$ or $SF_6$-gas at a high pressure ~10–20 atmos.

The *ion source* consists of an evacuated discharge tube $A$ that can be raised to potentials ~ 10 MeV. It is placed inside the dome at the high voltage end so that ions are accelerated from the high voltage to the low voltage end of the tube, where accumulated particles strike the target $T$ which is earthed.

**Uses and merit :** van de Graaff generator can be used to accelerate protons, deuterons etc. up to an energy of about 12 MeV beyond which the air-insulation breaks down. It is also used to accelerate electrons by a reversal of the potential of spray-voltage and using a hot filament as the source of electrons. An *important feature* of van de Graaff generator is the *constancy of voltage* to within 1%. Another *distinctive feature* is the *sensitivity* and *control of voltage.* The terminal potential is controlled in most generators by regulating the corona discharge current. van de Graaff generators provide good quality beam of high intensity having good energy-resolution and are extensively used in nuclear research.

## 5.3 Tandem accelerator

A ***Tandem electrostatic accelerator*** is an improvement over the usual single stage van de Graaff generator. It accelerates singly charged positive ions to twice the energy corresponding to the energy of the HV-terminal.

The *principle* of operation of the machine can be understood from Fig. 5.2.

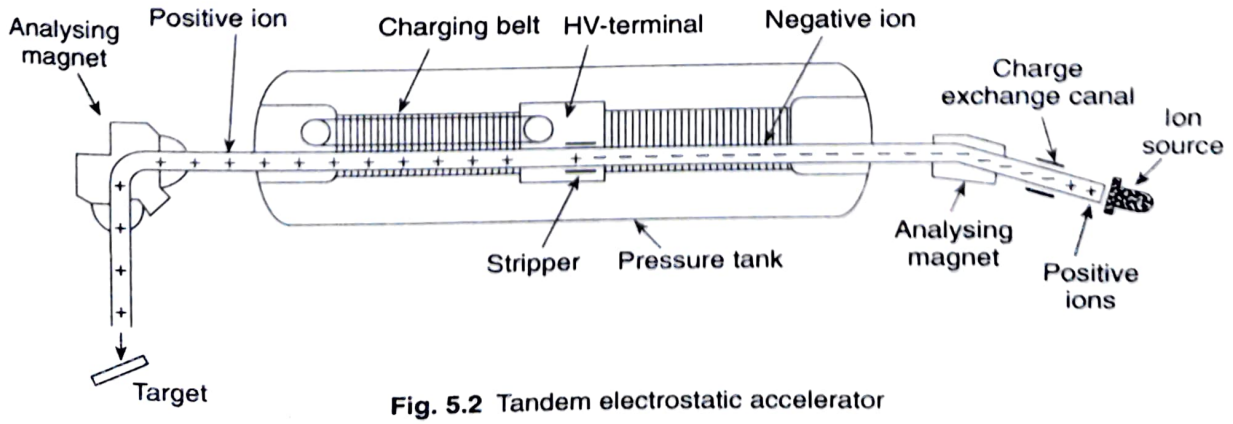


**Fig. 5.2** Tandem electrostatic accelerator

In tandem accelerator, the *HV-terminal* is at the centre of a long *pressure tank* so that the ion source and the target at the opposite ends are both at the ground potential. The *positive ions* from the *ion source* are accelerated and pass through the *charge exchange canal* where a flow of hydrogen adds two electrons successively to them so that they get transformed to *negative ions* with unit charge. These are then accelerated towards the positive HV-terminal. At the positive terminal, their kinetic energy becomes $V$ eV, where $V$ is the voltage (in volt) on the terminal.

When the ions are still inside the positive terminal, they enter a gas-containing canal, called *electron stripper*, and are stripped of their electrons to become positive ions, without change of energy. The positive ions, out of the stripper, have a distribution of charge states and pass through a *magnetic sector*, the parameters of which are set to select ions only of a particular charge state, say $+ne$. The positive ions are now accelerated in the accelerating column, housed in the pressure tank along with the HV-terminal. The accelerated positive ions finally pass through *analysing magnet* to hit the *target*.

***Total energy gained*** by a positive ion of charge state $+ne$ is

$$E = 1 \text{ eV} + n \text{ eV} = \mathbf{(1 + n)\ eV}$$

where $V$ is the potential of HV-terminal.

• If the positive ion is also singly charged, the total energy $E = (1 + 1)\,\text{eV} = 2\text{eV}$, that is simply doubled.

## 5.4 Linear accelerator or LINAC

The *linear resonance accelerator* or ***linear accelerator*** (also called ***linac***), is a device that successfully accelerates charged particles, including heavy ions, along a straight line in multiple steps by an oscillating electric field. It was developed by Louis Alvarez and co-workers.

The *basic principle* of a *linear accelerator* is to increase the velocity of charged particles in *successive steps* along a *straight line* (hence the name) when they are subjected repeatedly to an accelerating electric field. The particles in a linear accelerator are energized by a series of linear pulses at radio-frequencies, that is, by extra pushes at the right moments of time (*correct phase*).

**Apparatus** — Fig. 5.3 shows a schematic diagram of a linear accelerator. It consists essentially of a long tube $LT$, evacuated to a high degree, inside which a series of *coaxial hollow* metal cylinders $C_1, C_2, C_3, \cdots$ etc., called *drift tubes*, are arranged.

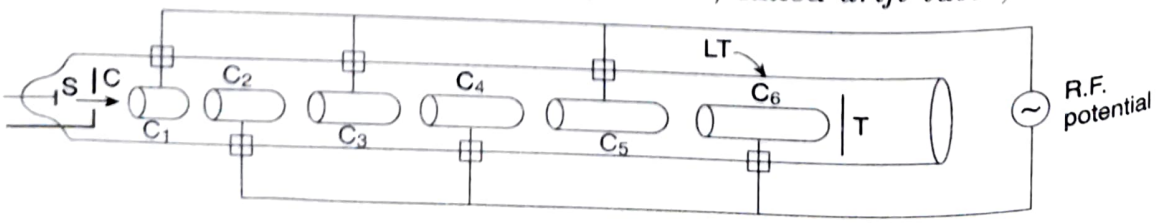


**Fig. 5.3** Linear accelerator

At one end of the main tube is placed the ion source $S$ where positive ions are produced. The ions are collimated by a collimator $C$ along the axis of the accelerator so that they pass through the drift tubes. The alternate drift tubes are in electric contact and the *two sets* of tubes so formed — *odd* and *even* — are connected to a high frequency (~10 megcycle) *RF*-oscillator. At any instant, the successive tubes have opposite polarity. The peak ac voltage is about 40 kV. The reversal of potential takes place periodically, according to the frequency of alternation of the oscillator. The ions in their passage through the drift tubes are accelerated in steps to leave finally the last drift tube to impinge on the target $T$ placed at the other end of the main tube. As the frequency of the oscillator is fixed, each succeeding drift tube is *greater in length* than its preceding one by a *calculated amount* to compensate for the increasing speed of positive ions.

**Principle of action** — The charged ions get accelerated *only in the gaps* between two adjacent cylinders where they are acted upon by the electric field present. But, inside the drift tubes, they travel with uniform velocities as there is no field inside.

If a positive ion leaves the collimator $C$ at a time when the first cylinder $C_1$ has a negative potential, the ion will be accelerated across the gap but travel with constant velocity in $C_1$. When the ion comes out of $C_1$ and if it is positive w.r.t. $C_2$, it will again be accelerated across the gap between $C_1$ and $C_2$, and its velocity will further increase. If $q$ be the charge on the ion, the increment in energy due to acceleration would be $qV$ where $V$ is the potential. In this way, in every gap between two cylinders, there will be successive accelerations, provided the ion always enters the field on leaving a cylinder, exactly when the potential is changing favourably. This ***matching*** is ***critical*** and is achieved only if the *time taken by the ion to traverse a cylinder is exactly equal to half the time period of rf-oscillator.*

If $l_n$ be the length of the $n$th cylinder, the time $t_n$ taken by an ion to travel through it is given by

$$t_n = l_n / v_n \tag{5.4.1}$$

where $v_n$ is the velocity of the ion inside the $n$th cylinder. So, the ***condition for synchronization*** is

$$t_n = \frac{l_n}{v_n} = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2}\frac{\lambda}{c} \tag{5.4.2}$$

where $T$ and $f$ are respectively the time period and the frequency of the rf-oscillator, $\lambda$ the wavelength of rf-waves and $c$ the velocity of light in vacuo.

**Drift tube length and energy of an ion** — Let an ion of charge $q$ be accelerated through the $n$th drift tube at a time when the rf-oscillator provides maximum potential $V_m$ across the gap.

$$\therefore \quad \text{Energy imparted to the ion} = nqV_m$$

since the ion has undergone $n$-accelerations in the preceding $n$ gaps and each acceleration gives rise to an energy-increment $qV_m$.

$$\therefore \quad nqV_m + K = \tfrac{1}{2}mv_n^2 \quad (\textbf{\textit{non-relativistic}}) \tag{5.4.3}$$

where $m$ is the mass of the ion, and $v_n$ its velocity inside the $n$th drift tube and $K$ its initial kinetic energy when it entered the first drift tube.

$$\therefore \quad \boxed{v_n = \sqrt{\frac{2nqV_m}{m}}}, \tag{5.4.4}$$

ignoring $K$.

With this value of $v_n$, we get from (5.4.2),

$$l_n = \frac{v_n}{2f} = \frac{1}{2f}\sqrt{\frac{2nqV_m}{m}} = \frac{1}{f}\sqrt{\frac{qV_m}{2m}}\sqrt{n} \tag{5.4.5}$$

The *lengths of the drift tubes should therefore be proportional to the square root of natural numbers*, i.e. in the ratio $1 : \sqrt{2} : \sqrt{3} \cdots$ etc.

It can be shown that the total length of the accelerating column, i.e., the sum of lengths of the drift tubes, ignoring the lengths of the gaps and the initial energy, is given by

$$L = \frac{1}{2f}\left(\frac{2qV_m}{m}\right)^{1/2} \sum_{1}^{n} \sqrt{n} \tag{5.4.6}$$

• Once the ions have attained velocities close to the velocity of light (relativistic), the lengths of the drift tubes become almost constant.

**Usefulness** — The linear accelerators are useful for accelerating protons and other heavier ions to energies of about 32 MeV.

The main ***advantage*** of a linear accelerator is that here *no magnet* is necessary to guide the particles. They emerge from the machine automatically as an excellent well-collimated beam and hit the target directly, unlike the circular type of machines like cyclotron (see later) where the emergent beam spreads out. Moreover, in linear accelerators, the emergent beam is *highly intense* and *homogeneous*. Since again the acceleration takes place in a straight line, the *radiation loss is much less* unlike the circular machines.

It is however ***not possible*** to accelerate ***electrons*** to very high energies in such machines as the electrons become relativistic even at few MeV and start moving out of phase. Also, the linacs require ultra-high-frequency oscillator for synchronization.

• The design of linear accelerator meant for the electrons however is different, consisting of tubes nearly equal in length, essentially of a *continuous tube*, down which an electromagnetic wave propagates. The tube is rather a ***wave guide,*** as it contains apertures deployed at intervals determined by the frequency of the travelling wave and also the size of the tube.

• The size of the linac depends on the final energy and the particle type. The *electron linac* at Stanford in USA is over 3 km in length and can accelerate electrons to 50 GeV.

• Many hospitals in our country have now-a-days 10 MeV electron linacs that produce photons to irradiate malignant tumours.

## 5.5 Cyclotron

The ***cyclotron***, first devised in 1931 by Lawrence and Livingstone, is a device to speed up charged particles in successive steps, along circular or rather spiral trajectories, under the combined influences of electric and magnetic fields.

**Construction** — It consists essentially of *two flat semi-circular hollow metal chambers* $D_1$ and $D_2$, called the *dees* (D) because of their shapes (Fig. 5.4). These dees are placed with their diametric edges *parallel* and *slightly separated* from each other so as to produce a narrow gap between them. The entire set-up, i.e., the dees, ion source etc. are placed in a highly evacuated chamber or vacuum tank $C$ and are connected to an rf-oscillator so that a *high frequency* (10 to 20 MHz) alternating potential is applied between the dees acting as electrodes. The potential between the dees thus alternates rapidly and the electric field in the gap is first directed to one dee and then to the other. The space within each dee is however *field-free* (zero field). The chamber $C$ is mounted *horizontally* between the pole pieces of a *huge electromagnet* that provides an *intense* and *uniform vertical* magnetic field of several Tesla. Since the field must be

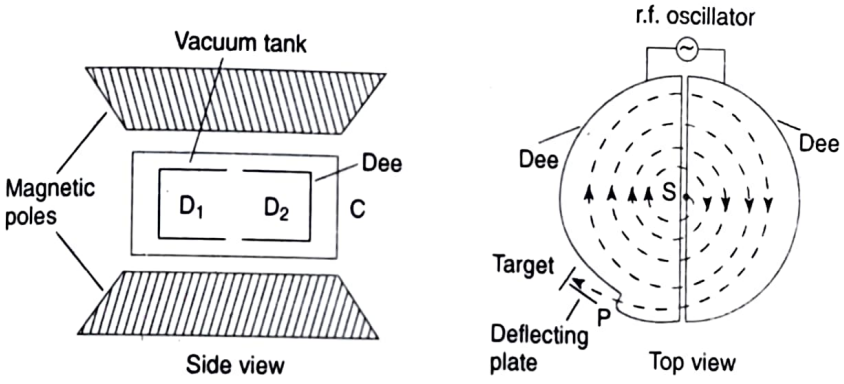


**Fig. 5.4** Cyclotron

uniform over the whole of the dees, the diameter of the poles should be slightly greater than that of the dees.

The ion-source $S$ is located near the mid-point of the gap between the dees. The ions, after being accelerated, are brought *out of the chamber* through a *window* $P$ by a *charged* deflecting *plate* to eventually bombard the *target*.

**Principle of action** — Let a positive ion of charge $+q$ leave the ion-source between the dees and enter the negative dee at the instant with velocity $\vec{v}_0$, normal to the magnetic lines of force. Due to the magnetic field $\vec{B}$, the force $\vec{F}$ acting on the particle (ion) is given by

$$\vec{F} = q\vec{v}_0 \times \vec{B} \tag{5.5.1}$$

But the magnetic field $\vec{B}$ is perpendicular to the direction of the force $F$ so that

$$F = qv_0B \tag{5.5.2}$$

Under the action of the field, the path of the particle (ion) is bent into a circle of radius $r_0$, given by

$$qv_0B = \frac{mv_0^2}{r_0} \quad (\textit{resonance condition})$$

where $mv_0^2/r_0$ is the centripetal force to the particle.

$$\Rightarrow \quad r_0 = \frac{mv_0}{qB} \tag{5.5.3}$$

where $m$ is the mass of the ion (particle).

When the particle is inside the dee, its speed stays constant; but as it leaves the dee to reach the gap, after describing the semicircular path inside the dee, the electric field of the rf-source changes direction *synchronously*. The particle thus gets a 'kick' and is accelerated into the other dee where, because of its increased speed $v$, it moves in a semi-circular path of comparatively larger radius $r$, given by

$$r = \frac{mv}{qB} \tag{5.5.4}$$

to emerge out again into the gap between the two dees. So, the particles spiral steadily outwards in the dees.

The ***frequency of revolution*** of the particle in the circular orbit is given by

$$\boldsymbol{f = \frac{v}{2\pi r} = \frac{vqB}{2\pi mv} = \left(\frac{q}{m}\right)\frac{B}{2\pi}}, \quad \text{using (5.5.4).} \tag{5.5.5}$$

This is known as the ***cyclotron frequency*** or ***cyclotron resonance frequency***. It is also the frequency of the a.c. oscillator.

The *important fact* that emerges from (5.5.5) is that *the frequency of revolution is independent of the speed of the ion and the radius of the path* in the dees. So, if the electric field reverses regularly at a frequency $f'$, exactly equal to $f$, the field in the gap is always in the right direction to accelerate a charged particle by an impulse at each $D$-crossing.

$$\therefore \quad \boxed{\left(\frac{q}{m}\right)\frac{B}{2\pi} = f = f'} \tag{5.5.6}$$

This is called the ***basic resonance equation*** *for a fixed frequency cyclotron.*

Thus, the revolving charged particle is steadily speeded up describing a *flat spiral* of increasing radius. Finally, it reaches the periphery of the dees and is brought out of the chamber through the window by a negatively charged deflector to impinge on the properly mounted target.

**Energy of particle** — Kinetic energy $E_k$ of the charged particle is $E_k = \frac{1}{2}mv^2$.

$\therefore$ The maximum kinetic energy of the particles when they leave the cyclotron is

$$E_m = \tfrac{1}{2}mv_m^2 = \tfrac{1}{2}m\left(\frac{r_m qB}{m}\right)^2, \quad \text{using (5.5.4).}$$

$$\therefore \quad \boxed{E_m = \frac{q^2B^2}{2m}r_m^2} \tag{5.5.7}$$

where $r_m$ = the maximum radius of the trajectory.

Thus the *maximum energy* that can be imparted to a particle in a cyclotron is directly proportional to (i) the *square* of *radius of the dees* ($E_m \propto r_m^2$) and (ii) the *square* of the *magnetic field strength* ($E_m \propto B^2$) of the magnet.

• The maximum energy of the particles expressed in equivalent voltage $V$ is given by

$$qV = E_m \Rightarrow V = qB^2 r_m^2/2m \tag{5.5.8}$$

• *An alternative expression* for $E_m$ can be obtained on elimination of $B$ from (5.5.7), using (5.5.6). We shall then get

$$\therefore \quad \boldsymbol{E_m = 2m(\pi f r_m)^2} \tag{5.5.9}$$

**Usefulness and limitations** — The cyclotron can energise protons, deuterons, $\alpha$-particles to 10-50 MeV. Its ***main advantage*** is that it does *not* demand any excessively high voltage source for producing high energy particles. Further, *no accelerating tube* capable of withstanding high potential gradients is necessary. It is also smaller in size compared to linac.

The ***most serious limitation*** imposed on the action of a cyclotron is however due to the *relativistic increase in mass* of the accelerated particle, given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \tag{5.5.10}$$

where $m_0$ is the rest mass of the particle and $m$ its moving mass at $v$.

∴ The frequency of revolution of the particle

$$f = \left(\frac{q}{m}\right)\frac{B}{2\pi} = \left(\frac{q}{m}\right)\frac{Bc^2}{2\pi c^2} = \frac{Bqc^2}{2\pi(m_0c^2 + E_k)} \tag{5.5.11}$$

where $E_k$ is the kinetic energy of the particle corresponding to the speed $v$.

So, as the mass or speed of the particle increases for a fixed $B$ and $q$, frequency $f$ decreases. Thus in a fixed frequency cyclotron the particle, as it spirals out from the source, will lag in phase behind the accelerating dee voltage. It would thus cease to get accelerated. The frequency of oscillation must be suitably modified (reduced) to ensure *resonance acceleration* in the gaps. This cannot be done without upsetting the condition for low energy particles. *With electrons, the relativistic change in mass is more pronounced than with positive ions, even for low energies. This makes the* ***cyclotron rather impractical for accelerating electrons to high energies.***

The ***other limitation*** is that the limit to the maximum energy in a cyclotron is set by the engineering difficulties and enormous expenses. Also, from (5.5.7), the energy $E_k \propto r^2$, for a given particle so that the size of a cyclotron increases more rapidly than the corresponding increase in energy.

## 5.6 Compensation for relativistic mass gain

There are however ways to overcome the limitation imposed on the action of a cyclotron due to the relativistic mass-variation by proper compensation for relativistic mass gain.

is needed for supplying the variable flux to accelerate the electrons, (ii) an accelerated electron loses energy by radiation and the rate of radiation increases, according to classical electrodynamics, with the fourth power of the energy so that a limit is set for the high energy electrons by *radiation loss* and (iii) in the practical condition, the maximum energy that can be imparted to an electron is about 1000 MeV or 1 BeV.

**Cyclotron vs. Betatron**

| Cyclotron | Betatron |
|---|---|
| 1. Fixed-frequency cyclotron employs no alternating magnetic field; field is *constant.* | 1. Betatron, however, employs an oscillating field which is called the *induction field.* |
| 2. Particles are accelerated in *helical orbits* and orbital radius goes on increasing and equals the radius of the dees in the limit. | 2. The acceleration of the particles in a betatron takes place in *a circular orbit* which is essentially of constant radius. |
| 3. A fixed-frequency cyclotron is not suitable for accelerating electrons. | 3. A betatron is specifically meant for accelerating the electrons. |
| 4. Cyclotron suffers from the problem of orbital stability. | 4. It also suffers from the problem of orbital stability. |
| 5. This cannot be used as an X-ray generator. | 5. This is a very highly efficient X-ray generator. |

• Note that the final energy $E_T$ given by (5.9.10) is *independent* of $\omega$, the angular frequency of the changing flux. However, $\omega$ should not be too small, for otherwise the time of acceleration is increased and the radiation loss (ignored to derive (5.7.10)) of energy also increases.

## 5.8 Synchrocyclotron

We have already seen that the relativistic mass-increase can be compensated to save synchronization by keeping the magnetic field constant and decreasing the frequency of electric field so that, with increase of velocity of the particle, the frequency

$$f_0 = \frac{qB}{2\pi m_0}\sqrt{1 - v^2/c^2}$$

is always constant. The orbital frequency of the particle is thus made synchronous with the frequency of the alternating field. Such ***an accelerator in which the frequency of the electric field is modulated to obtain a phase stable orbit is called frequency modulated (FM) cyclotron or synchrocyclotron.***

**Construction** — The basic design of synchrocyclotron is similar to that of a cyclotron. But it consists of only *one* dee (Fig. 5.7), unlike the cyclotron. Fixed magnetic field is applied with the help of pole pieces of an electromagnet in a plane normal to

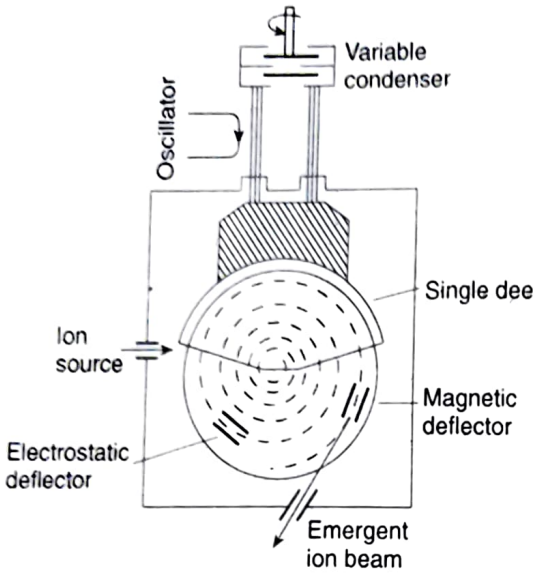


**Fig. 5.7** Synchrocyclotron

that of the dee. The frequency modulated alternating potential is applied between the dee and the earth. The particles like protons are injected from a ion source and are accelerated by the alternating electric field in correct phase to get deflected by the magnetic field along circular paths of increasing radii. The particles are accelerated whenever they enter or leave the dee. The frequency of the electric field is changed *continuously and slowly* so that the phase stability is not disturbed, by rotating the movable vanes of a variable capacitor *slowly*. At the outer edge of the dee, the energised particles are deflected out of the field by the electrostatic or magnetic deflector to hit the target.

There is a *disadvantage* in a synchrocyclotron in that the output beam current is relatively small, for only a small fraction of the ions is captured in the phase stable orbit of maximum radius and energy.

**Computation of fractional frequency shift** — The maximum frequency of the oscillator is designed to be the one corresponding to the value of $f$ near the ion source at the centre. Call it $f_c$. At the centre, the kinetic energy of the particle $E_c$ is small and $B = B_c$. Let $E_k$ be the kinetic energy of the particle as it reaches the dee-radius where $B$ has the value $B_c - \Delta B$. Also, let the frequency of the oscillator output be reduced to $f_c - \Delta f$ so that it is matched throughout between the frequency of the a.c. voltage and that of the circulating ion.

Now, we have
$$f_c = \frac{1}{2\pi}\frac{B_c q c^2}{m_0 c^2} \tag{5.8.1}$$
using the relation (5.5.12), since $E_c \ll m_0c^2$ at the centre of the dee.

And,
$$f_c - \Delta f = \frac{1}{2\pi}\frac{(B_c - \Delta B)qc^2}{(m_0c^2 + E_k)} \tag{5.8.2}$$

$$\therefore \quad E_k = \frac{qc^2}{2\pi}\frac{(B_c - \Delta B)}{(f_c - \Delta f)} - m_0c^2$$

$$= \frac{qc^2}{2\pi}\left\{\frac{B_c - \Delta B}{f_c - \Delta f} - \frac{B_c}{f_c}\right\}, \quad \text{using (5.8.1)}$$

$$\simeq \frac{qc^2}{2\pi f_c^2}(B_c\Delta f - f_c\Delta B) = \frac{qc^2 B_c}{2\pi f_c}\left(\frac{\Delta f}{f_c} - \frac{\Delta B}{B_c}\right)$$

$$= m_0c^2\left(\frac{\Delta f}{f_c} - \frac{\Delta B}{B_c}\right), \quad \text{using (5.8.1).}$$

$$\therefore \quad \boxed{\frac{\Delta f}{f_c} = \frac{E_k}{m_0 c^2} + \frac{\Delta B}{B_c}} \tag{5.8.3}$$

This relation (5.8.3) gives the *fractional frequency shift* required to impart a final energy $E_k$ to the charged particle.

If $V_0$ be the peak accelerating voltage, then at the dee-crossing the particle has its energy increased by an amount $\Delta E$ given by

$$\Delta E = qV_0 \sin \phi_0$$

where $\phi_0$ is called the *synchronous phase angle* and is measured from the cross-over point A (Fig. 5.5).

• Synchocyclotron is not built any longer although they can be scaled up to any energy. This is because the synchrotron is a more versatile mechine at high energies.

## 5.9 Synchrotrons

To accelerate particles to increasingly higher energies by cyclotron or syncho-cyclotron, one is to build machines with increasingly larger magnets. And magnet is the costliest part—prohibitibly costly—of a cyclotron. The situation can be overcome in ***synchrotron*** where both the magnetic field and the resonance frequency vary simultaneously. As the energy is increased, the particle orbit is very nearly of a constant radius at all the energies. And the magnetic field needs to be applied only at the circumference and not throughout the entire circular volume as in a cyclotron. The particles are confined to circular *paths* by *annular magnets.*

### 5.9.1 Electron synchrotron

The disadvantages of a betatron in accelerating electrons are greatly eliminated in *electron synchrotron* which, in fact, combines the ***accelerating system*** of the cyclotron with the *ring-shaped pulsating guide field* of a betatron.

**Construction** — The essential parts of an ***electron synchrotron*** (Fig. 5.8) are (i) a *doughnut-shaped toroidal vacuum chamber* or *tube* into which are injected electrons, initially accelerated by a linear accelerator to an energy of about 50-100 keV when their speed is about $0.5c$, (ii) the *magnet* which, unlike, the massive magnets of betatron, has *annular* (ring-shaped) *pole pieces* that closely follow the outline of the doughnut to guide the electrons into an equilibrium orbit inside the doughnut, (iii) *soft-iron flux bars*, located in the central gap and serving as the central magnetic core to start the machine as a betatron and (iv) copper or silver coating in a part of the interior of the doughnut to give a $D$(dee)-shaped *resonance cavity* and is connected to a rf-*oscillator*.

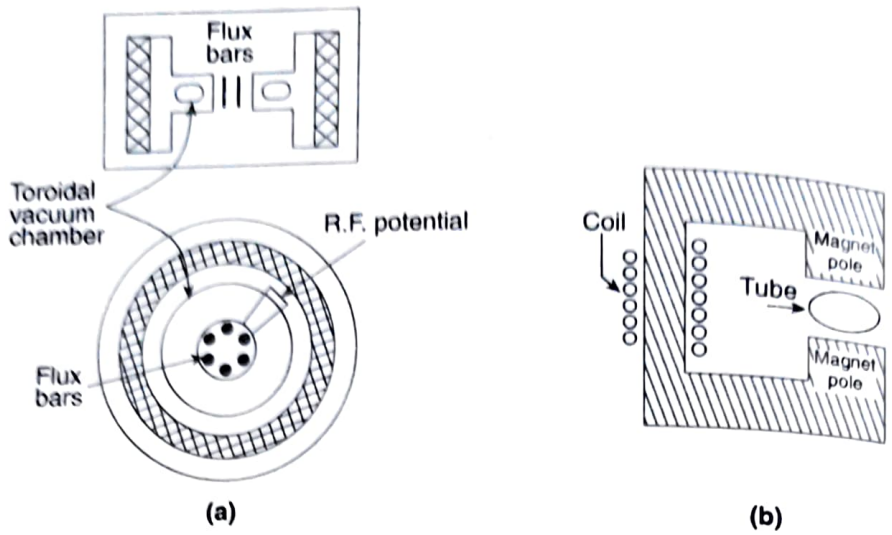


**Fig. 5.8** Electron synchrotron

**Action** — The performance of the machine occurs in *two phases*. In the *first phase* the electrons are accelerated to energies of about 1–2 MeV by the varying magnetic flux as in a betatron. As the flux varies, the electrons move in circular orbits and get accelerated. At energies of about 2 MeV, the flux bars become *magnetically saturated* and cannot accelerate the electrons further. The *betatron action* thus *ceases* and the first phase of the machine is over.

In the *second phase*, the R.F. oscillator becomes active. If the potential applied to the resonance cavity is at the proper frequency, the electrons resonate and are accelerated periodically, as in a cyclotron, at the beginning and end of the $D$. When the electrons are accelerated to the maximum energy, the oscillator is suddenly switched off to deflect the electrons outward towards the target. The energy acquired is as high as 1000 MeV or 1 BeV. Using tungsten target, very hard X-rays of energy about 300 MeV have been produced.

The machine however suffers from a *limitation* imposed on the energies of the electrons due to the *radiation loss.*

**Energy attained** — The energy attained by the electrons may be calculated readily from the expression

$$E = \sqrt{p^2c^2 + m_0^2c^4} = E_k + m_0c^2 \tag{5.9.1}$$

where $p$ is the momentum and $E_k$ the kinetic energy of the electron.

If $p$ and $E_k$ are very large compared to the rest mass energy $m_0c^2$, then from equation (5.9.1) above, we have

$$E_k \simeq pc \simeq Berc \tag{5.9.2}$$

where $r$ is the equilibrium orbit radius and $B$ the magnetic flux density.

If $B$ is expressed in $\text{Wb/m}^2$, $r$ in metre and the energy in MeV, then from equation (5.9.2), we obtain

$$E_k = 300Br \text{ MeV}$$

The condition for a stable orbit, as in betatron : $\phi = 2\pi r^2 B$, must still be fulfilled. The use of a very heavy magnet over the entire volume of the vacuum chamber is *no longer needed.*

• The accelerating system is usually a metallic quarter-wave resonant cavity at electron-orbital frequency. The resonant cavity is formed by coating the inside and the outside of an angular sector of the doughnut-shaped vacuum chamber with metallic silver or copper.

### 5.9.2 Proton synchrotron (Bevatron)

**Construction and action** — The ***proton synchrotron,*** also called ***bevatron,*** essentially follows the same principle of construction and operation as electron synchrotron except that it is *designed to accelerate protons* or *heavy positive ions* that become relativistic at much higher energies. It has a race track design. The proton revolves in *stable orbits* of constant radius in a *doughnut-shaped vacuum chamber.* It has also *no solid magnetic core.* A ring-shaped magnet around the periphery produces a magnetic field normal to the chamber for bending the path of the particles. The magnet has four quadrants (four dipole sections) covering the doughnut (Fig. 5.9).

The protons to be accelerated are produced by an ion-source and are injected into the synchrotron orbit in the doughnut at low energy from a linac, in the form of periodic pulses. The protons are accelerated by an alternating rf-electric field, and the magnetic field is applied normal to the chamber from the ring-shaped magnet that can be excited

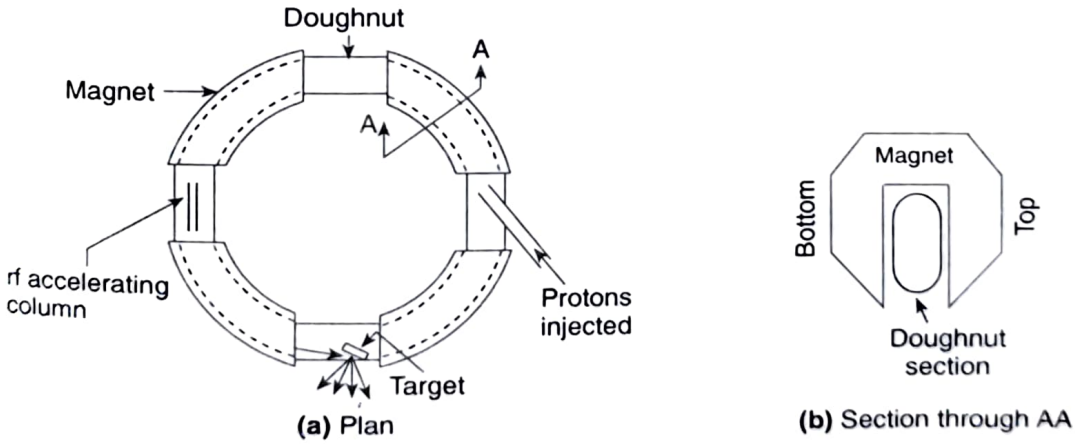


**Fig. 5.9** Bevatron or proton synchrotron

periodically. In one of the field-free straight parts, a high frequency resonator cavity is used with an increasing frequency corresponding to increasing speed of protons.

Just as with electrons, the guide magnetic field must steadily increase for each pulse of protons to prevent them from spiralling outward. The frequency of the electric field cannot however be kept constant as in electron synchrotron, but is to be *simultaneously* increased suitably to have a *resonance* with the orbital frequency of the ions. Thus the protons get accelerated in a *circular trajectory* of *constant radius* in successive steps

during the time when the magnetic field is increasing until the maximum energy is acquired. The frequency of the rf-field is suddenly changed, the orbit expands and the protons are ejected so as to hit a specific target.

**Calculation of frequency, energy and orbit-radius** — If the straight section were absent, the circulation frequency of protons of energy $E_k$ in equilibrium orbit of radius $r_0$ in a magnetic field $B$ would be given by (5.5.11) :

$$f' = \frac{Bqc^2}{2\pi(m_pc^2 + E_k)}$$

where $m_p$ is the rest mass of the proton and $q$ its electric charge.

Because of the straight section, the circulation frequency would be modified to

$$f' = \frac{Bqc^2}{2\pi(m_pc^2 + E_k)} \cdot \frac{2\pi r_0}{2\pi r_0 + 4L}$$

$$= \frac{Bqc^2}{m_pc^2 + E_k} \cdot \frac{r_0}{2\pi r_0 + 4L} \tag{5.9.3}$$

where $L$ is the length of each of the four straight sections.

The kinetic energy $E_k$ of a proton of momentum $p$ may be written as

$$E = E_k + m_pc^2 = \sqrt{p^2c^2 + m_p^2c^4} = \sqrt{(Bqr_0c)^2 + m_p^2c^4} \tag{5.9.4}$$

where $E$ is the total energy and $p = Bqr_0$. So, from (5.9.2), we have

$$E_k^2 + 2E_km_pc^2 = (Bqr_0c)^2$$

$$\text{or,} \quad E_k(E_k + 2m_pc^2) = (Bqr_0c)^2 \tag{5.9.5}$$

$$\Rightarrow \quad r_0 = \frac{\{E_k(E_k + 2m_pc^2)\}^{\frac{1}{2}}}{Bqc}. \tag{5.9.6}$$

If $B$ be in Tesla, and $E_k$ and $m_pc^2$ in GeV, then the $r_0$ above is given in metre.

So, from (5.9.6),

$$r_0 = \frac{\{E_k(E_k + 2m_pc^2\}^{\frac{1}{2}}}{0.3B} \text{ m} \tag{5.9.7}$$

$$\text{and,} \quad B = \frac{\{E_k(E_k + 2m_pc^2)\}^{\frac{1}{2}}}{0.3r_0} \text{ Tesla} \tag{5.9.8}$$

This shows how to increase the magnetic field $B$ at the equilibrium orbit with increase in proton energy $E_k$, as it circulates in the orbit.

Substituting for $B$ in (5.9.3), we obtain

$$f' = \frac{c\{E_k(E_k + 2m_pc^2)\}^{\frac{1}{2}}}{(m_pc^2 + E_k)(2\pi r_0 + 4L)}. \tag{5.9.9}$$

If the accelerating voltage is to be in phase with $f'$, its rf-value should always match $f'$ and vary according to the variation of $f'$ with $E_k$.

• A proton synchrotron can energize protons to values as high as 10 BeV. As the machine is capable of energizing particles to the order of BeV, it is properly termed as a ***bevatron***. It is also called a ***cosmotron***.

### 5.9.3 Alternating gradient (AG) synchrotron

In proton synchrotron, the cross-section of the vacuum chamber must be large enough to accommodate the oscillations of particles. If not, the particles will hit the sides of the chamber and get lost. This again dictates that the size and the gap of the magnet should also be large so as to accommodate the vacuum chamber. Such magnets with a constant gradient for the field is prohibitively costly. In fact, the maximum energy attainable in a synchrotron is limited by such factors as *increased size, high cost of the ring magnet* and the *difficulties in controlling the stray particles* and *achieving a strong focussing*.

The above difficulties have been done away with in the ***alternating gradient synchrotron***, also called ***strong focussing synchrotron***. The *basic principle* is the strong focussing of the beam, which will reduce the cross-section of the chamber and the size of the magnet, with the help of a number of alternate focussing and de-focussing (i.e., converging and diverging) ion lenses along the circular path of the ion. The net effect is that of focussing to keep the particle in a stable orbit (Fig. 5.10).

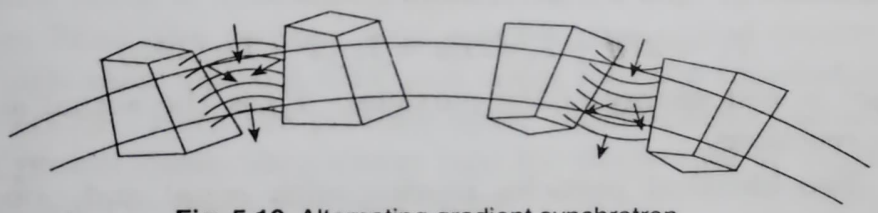

**Fig. 5.10** Alternating gradient synchrotron

A number of C-shaped magnets are deployed along the circular path such that the alternate magnets are *in opposition*, creating a gradient of magnetic field in the opposite direction. The pole pieces of the alternate pairs of magnets are so designed and distributed around the doughnut vacuum chamber that the field alternately increases and decreases in the radial direction. The beam thus experiences a strong focussing similar to that of light by a number of alternate converging and diverging lenses. This saves the cost and maintenance of the magnets.

The first accelerator of this type was designed and commissioned by CERN. Protons could be energized to a very high value, as high as about 30 GeV, in alternating gradient synchrotrons.

## 5.10 Storage ring and particle collider

A ***storage ring*** is essentially a ring-shaped vacuum chamber, kept between the pole-faces of a ring of magnets, in which a beam of very high energy charged particles from accelerators is injected and made to circulate *to store* it for a long time. The